Coupled Padé Approximation-Finite Element Method Applied to Microwave Device Design

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Abstract – In this paper, a fast and rigorous analysis method, combining a Padé approximation and a finite element method, is presented. The finite element method and the Padé approximation are first described. The method is then applied to the design of two microwave devices: a narrow band bandpass filter and a broadband microwave module. The accuracy of this approach is demonstrated by the good agreement between the coupled Padé approximation- finite element analyses and the standard finite element ones, and also between the coupled Padé approximation- FE analysis and the experimental response of filter.

I. Introduction

Recent advances in microwave computer aided design and increasing computer capabilities spur engineers on to use electromagnetic field solvers in order to design complex microwave devices.

The finite element method, used in the field of electromagnetism, enables to take into account the geometry details, the different domains, as far as the material point of view is considered. It has shown its accuracy but presents a great disadvantage in terms of computation time when the considered device includes a lot of discontinuities. As a matter of fact, one computation must be performed at each frequency point of the whole bandwidth. If the required accuracy of the frequency response is quite high, the computation time will increase significantly.

Several techniques have already been presented in order to analyse microwave devices, reducing the number of electromagnetic computations [1]-[4].

The method presented in this paper is based on Padé approximation and it enables to draw the entire performance envelop of the electrical performance. A single computation is necessary to analyse the electrical parameters in the entire bandwidth. Consequently, the electric behaviour of complex passive microwave components can be determined faster.

In the first part, the finite element method and the Padé approximation are presented. Then, the method is applied to design two microwave devices.

II. Finite Element Method

The finite element method [5],[6], is based on the fact that the geometry of the entity can be divided into sub-elements such as triangles in 2 dimensions or tetrahedrons in 3 dimensions.

In the case of the finite element method applied to electromagnetism, Maxwell equations are solved tetrahedron by tetrahedron in order to determine the electric field ($\vec{E}$ formulation) or the magnetic field ($\vec{H}$ formulation), taking into account the boundary conditions (electric wall, magnetic wall or a distributed access).

Applying the $\vec{E}$ formulation, the electric field is the solution of the equation (1), and the boundary conditions are defined by the following relations :

$$\begin{aligned} &\vec{J}_k = -\vec{n}_{p_k} \wedge \vec{H} \text{ on the distributed access} \\ &\vec{n}_e \wedge \vec{E} = \vec{0} \text{ on the electric walls} \\ &V = -\int \vec{E}.\vec{dl} \text{ on the localised access} \end{aligned} \qquad (2)$$

with: $\vec{E}$: electric field, $\vec{H}$: magnetic field, V : voltage, $\vec{n}_{p_k}$: access normal vector, $\vec{n}_e$: electric wall normal vector.

The system is solved either in free oscillations or in forced ones. In the case of free oscillations, the second member of the equation equals zero by making a short-circuit at the level of the input and output port (no signal sources). The resolution of the eigenvalues of the system enables to know the repartition of the electromagnetic fields, to compute the unloaded quality factor and the resonant frequency. In the second case with forced oscillations, the

$$\iiint_V (\mu_i^{-1}\vec{rot}\vec{E})\,\vec{rot}\vec{\phi}\;dV - k_0^2 \iiint_V \varepsilon_i \vec{E}\vec{\phi}\;dV = -j\omega\mu_0 \sum_{k=1}^{n} \iint_{Sp_k} \vec{J}_k.\vec{\phi}\;dSp_k - j\omega\mu_0 \sum_{p=1}^{m} \int_{l_p} \vec{I}_p\;dl_p \qquad (1)$$

with : $k_0^2 = \omega^2\varepsilon_0\mu_0$, V : volume of the structure, $\vec{\phi}$: test function vector, $\vec{E}$: electric field,
n : number of modes in distributed accesses, Sp_k : surface of the distributed access, $\vec{J}_k$: surface distribution of the current,
m : number of localised access, l_p : contour of the localised access p, $\vec{I}_p$: line distribution of the current

second member is different from zero and the propagate and evanescent modes can be computed thanks to a modal decomposition.

With forced oscillations, the resolution of this electromagnetic problem is equivalent to solve a linear system with the following form :

$$A(f)X(f) = B(f) \qquad (3)$$

where f is the frequency

The first member of this equation corresponds to the first member of the equation (1) and the second member also corresponds to the second member of the equation (1). The elementary solutions $X_i(f)$ of this linear system enables to compute the electromagnetic field inside the device and also the different elements of the scattering matrix.

The matrix $A(f)$ can be expressed as :

$$A = K - \omega^2 M \qquad (4)$$

The matrices K and M in the domain containing no conductors, and characterised by a permittivity and a permeability independent of the frequency, are thus independent of the frequency. When the domain is a conducting one, two possibilities must be considered:

- The whole domain is conducting and can be defined by the Ohm law : $J = \sigma E$.

The relative permittivity is thus :

$$\varepsilon_r - i\frac{\sigma}{\omega\varepsilon_0} \qquad (5)$$

with ε_r, the relative permittivity of the domain which can also be complex or not.

- The domain is delimited by the surface which constitutes the frontier domain. The tangential electric field is $E_{tang} = Z_S J_S$, with Z_S the surface impedance that can be described by the following expression :

$$Z_S = \sqrt{\mu_0 \varepsilon_0^{-1}}\sqrt{\mu_r.(\varepsilon_r - i\frac{\sigma}{\omega\varepsilon_0})^{-1}} \qquad (6)$$

The matrix $B(f)$ can be expressed as:

$$B(f) = (B_0(f),...,B_n(f)) \qquad (7)$$

The second member characterises the current distribution on the access or port, which makes the link between the entity considered and the outside world. Four kinds of access must be taken into account :

- Distributed access : cutting of the structure whose reference waves are function of the frequency.
- Localised access : line whose voltage –current reference waves are independent of the frequency. The geometric entity corresponding to this localised access is thus a line whose length is negligible compared to the wavelength.
- Numeric access : surface belonging to the frontier of the structure.
- External access : this access corresponds to the input port and enables to define a plane input wave defined by its incident wave and its polarisation.

III. Parametrisation by Padé Approximation

In the standard approach, as the coefficient of the matrix A and B depends on the frequency, they must be computed for each frequency of interest. The method implemented by CADOE is based on a different approach using the same mesh for the geometry considered.

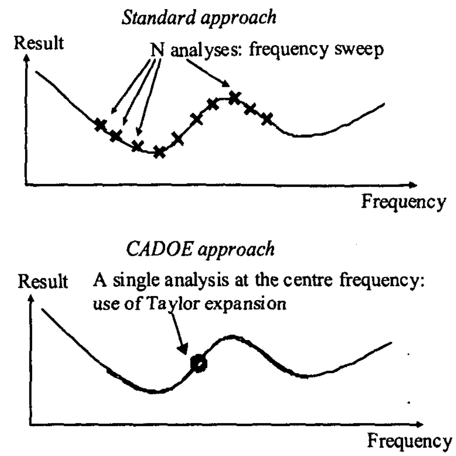

Fig.1. Padé approximation approach

The principle of this method is to build an explicit frequency formulation of the solution $X(f)$, based on the computation of a Taylor polynomial, or any other explicit function of the solution from the high order derivatives.

The problem to be solved is the following one :

$$A(f)X(f) = B(f) \qquad (8)$$

with $f \in [f_{min}, f_{max}]$.

We compute A, B and X at the centre frequency f_0.

The n-order derivative of the solution $X(f)$ can be deduced from the n-order derivative of equation (1):

$$\begin{cases} A(f)X^{(1)}(f) = B^{(1)}(f) - A^{(1)}(f)X(f) \\ A(f)X^{(m)}(f) = B^{(m)}(f) - \sum_{i=1}^{m} C_m^i A^{(i)}(f)X^{(m-i)}(f) \end{cases} \qquad (9)$$

The natural approach would consist in building the Taylor series of X :

$$X(f+\Delta f) \approx X(f) + X^{(1)}(f)\Delta f + ... + \frac{1}{m!}X^{(m)}(f)\Delta f^m \qquad (10)$$

Certain frequencies linked with the geometry and with the characteristics of the structure can be the poles (or singular points) of this approximation such as cut-off frequencies and resonant frequencies. Consequently, the Taylor approximation can only be used in a narrow bandwidth, delimited by the distance between the centre frequency and the nearest pole. Outside this bandwidth, the Taylor series does not converge. The use of Padé approximation enables to compute the approximation of $X(f)$ even at the singular points corresponding to the poles of $X(f)$:

$$X(f) \approx \frac{\sum P(f)}{\sum Q(f)} \tag{11}$$

The convergence radius of Padé developments is limited by the distance between the centre frequency and the first pole not included in $Q(f)$.

(11) can be described more precisely :

$$X(f_0 + \Delta f) \approx \frac{\sum_{i=0}^{n_p} P_i \Delta f^i}{\sum_{i=0}^{n_q} Q_i \Delta f^i} \tag{12}$$

The roots of $Q(f)$ are the poles of the function $X(f)$.

The polynomial $\Sigma P_i.\Delta f^{\,i}$ is evaluated for the resonant frequencies and gives the modes: ϕ_j. (f_j, ϕ_j) that form the modal basis solution of : $A(f)\phi=0$.

The CADOE method gives the response and the modal basis, whatever the complexity of the relation of the matrix A with the frequency.

Consequently, the following steps must be followed in order to analyse a structure with the parametrisation :

- Computation of the high-order derivatives
- Computation of the modal basis

For any frequency and for any excitation :

- Computation of $B_j(f)$
- Projection in the modal basis : $\phi_j^t B_j$
- Solution in the modal basis : X_j

IV. Application to Microwave Circuit Design

The finite element method coupled with the Padé approximation is applied to the design of two microwave devices.

A. Narrow band bandpass filter:

Narrow band bandpass filters are used in satellite communication systems, particularly in output multiplexers that combine different channel signals towards the emission antenna. Such a filter is often very sensitive to the geometrical dimensions of the structure. As a consequence, a theoretical study becomes necessary in order to predict the device behaviour, intending to replace costly dummy realisations. Particularly, the finite element method has shown its accuracy for the rigorous analysis of such complex structures.

The structure under study is the 5 pole filter presented in figure 2.The filter is composed of 3 cylindrical cavities excited on the TE_{113} dual mode with two input / output rectangular irises. The two polarisations of the dual mode are coupled using a screw at 45 degree angle from the polarisation axes in the two first cavities. In the third cavity, only one polarisation is coupled. Moreover, a coupling screw in each polarisation axis allows to tune the resonant frequencies.

The required centre frequency is 12.35 GHz with a 37.5 MHz bandwidth. The 5 pole filter has been designed applying an electromagnetic optimisation method [7] and all the electromagnetic analyses have been performed applying the finite element method coupled with the Padé approximation. For each analysis, a single computation at the centre frequency is necessary.

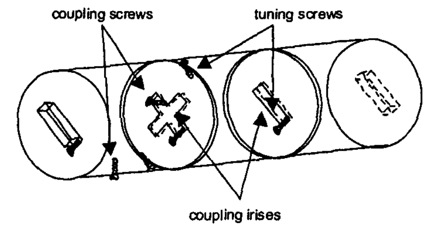

Fig.2. Narrow band bandpass filter

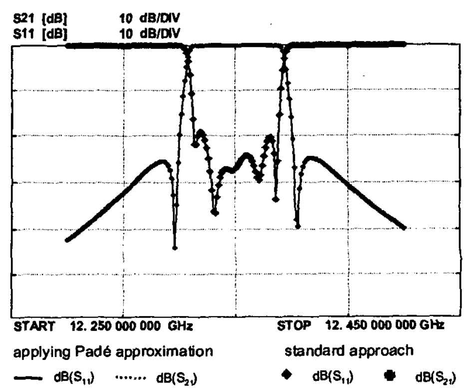

Fig.3. Compared electromagnetic analyses

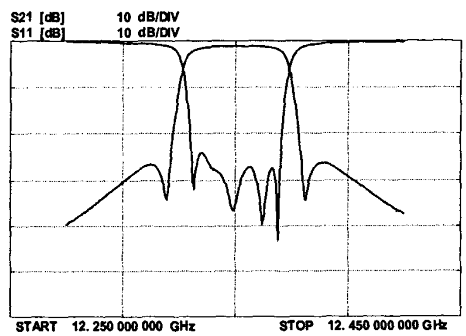

Fig.4. Experimental response

The final response computed applying the Padé approximation is compared in figure 3 with the response computed applying the standard approach.

The spectral analyses are in good agreement. The total CPU time applying the Padé approximation is 8 min in order to obtain the spectral response with 251 frequency samples on a HP9000/785 workstation. Applying the standard approach, the total CPU time is 2,5 min for each frequency, that is equivalent to 10,5 hours for 251 frequency samples.

The filter has been built and tested. The experimental response is presented in figure 4. The experimental results show a good agreement with the theoretical ones.

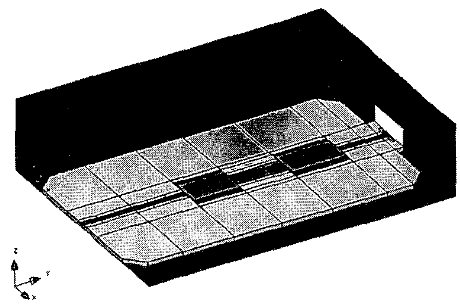

Fig.5. Microwave active module

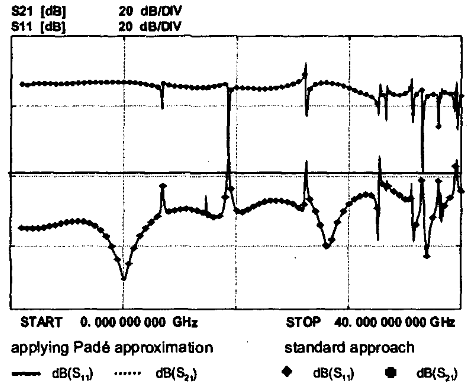

Fig.6. Compared electromagnetic analyses

B. Broadband microwave active module:

The design of highly integrated modules becomes more and more difficult when high operating frequencies are considered. An electromagnetic analysis is often necessary in order to predict indirect couplings and parasitic effects which may occur at microwave frequencies.

The structure under study is a microwave module, that is presented in figure 5. This module is composed of two MMICs connected together in a package.

The aim of this study is to characterise the circuit in its electromagnetic environment. The circuit is analysed, applying the finite element method coupled with the Padé approximation, in the whole frequency band, from 1-GHz to 40-GHz. Only three electromagnetic computations are necessary.

The final response computed applying the Padé approximation is compared in figure 6 with the response computed applying the standard approach. The spectral analyses are in good agreement. The total CPU time applying the Padé approximation is 75 min in order to obtain the spectral response with 401 frequency samples on a HP9000/785 workstation. Applying the standard approach, the total CPU time is 18 min for each frequency, that is equivalent to 120 hours for 401 frequency samples.

V. Conclusion

We have outlined a fast and rigorous analysis method combining a Padé approximation and a finite element method. This approach has been successfully applied to design two complex microwave devices. The efficiency of this method has been demonstrated comparing the Padé approximation results and the standard ones. Moreover, we have shown that this approach is completely consistent with an electromagnetic optimisation method, designing a five-pole narrow band bandpass filter in good agreement with the experimental test.

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